A New PAPR reduction Technique based on Precoding and Gamma Companding Technique for OFDM system

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Abstract

Multi-carrier modulation is less sensitive to time dispersion of the channel. In particular, Orthogonal Frequency Division Multiplexing (OFDM) has gained more attention in the last decade owing to rising power of digital signal processing. In spite of the several difficulties that entails during implementation of OFDM system, one of the major drawbacks in OFDM is high peak to average power ratio. Hence it is mandate to research on the characteristics of the PAPR in association with its distribution and reduction in OFDM systems. Myriad of techniques were introduced to reduce Bit error rate and PAPR. Amidst these techniques, precoding gives best solution for reducing PAPR. But in this technique, more null subcarriers are incorporated which leads to out-of-band power emission and in turn reduces the transmission rate. In order to minimize OOB power emission, we combine precoding in frequency domain and gamma companding technique in time domain. The gamma compander promotes OOB power emission. Simulation results reveals a trade off between OOB power emission and PAPR reduction.

Keywords: Orthogonal Frequency Division Multiplexing (OFDM), Peak to Average Power Ratio (PAPR), Bit Error Rate (BER), OOB power emission.

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1. Introduction

OFDM is a frequency division multiplexing method in which numerous carriers are sent in a single channel, the carriers are statistically independent to each other. Orthogonal frequency-division multiplexing (OFDM) is similar to coded OFDM (COFDM) and Discrete Multi-Tone modulation (DMT) [9/1],[5/2]. For carrying the data, closely-spaced sub-carriers which are orthogonally aligned are utilized. The data are divided parallel into channels with one for each sub-carrier. All the sub-carriers are modulated with any one of the modulation scheme with a very low symbol rate thereby maintaining the data rate almost like inherited single-carrier modulation schemes or the same bandwidth. In general, OFDM signal possess high Peak to Average Power Ratio by the side of the transmitter, which in turn affects the in-band distortion and the out-of-band radiation which occurs due to the nonlinearity of high power amplifier (HPA). This leads to increased bit error rate (BER). This has provoked the PAPR reduction as one of the most significant research area in OFDM systems. To mitigate this limitation, numerous techniques have been revealed by several researchers which include clipping [7/3], filtering, coding schemes, nonlinear companding transforms [5/2],[3/4], tone reservation and tone injection [7/3][8/5], Constellation shaping, partial transmission sequence and selective mapping (SLM) [2/6], [7/3], [8/5].

In this paper, we combine two PAPR reduction techniques the precoding schemes have been proposed for the effectiveness in reducing PAPR. Another scheme based on nonlinear companding transform such as gamma Companding this transform was applied after the IFFT process. The gamma compander cause spectrum side lobes generation in comparison with the original signal while reducing PAPR efficiently.

The remainder of this paper is organized as follows. Section II introduces the effect of PAPR in OFDM. Section III discusses the PAPR reduction technique in existence and section IV describes the proposed method to reduce the PAPR of OFDM systems. Section V provides the simulation report of proposed method. Finally, section VI gives the conclusion.

2. Problems in PAPR

PAPR is the ratio between the maximum power and average power of the data in a given OFDM symbol [6/7]. PAPR adversely increase the complexity of (AtoD)analog to digital and (DtoA)digital to analog converter which in turn affects the overall efficiency of the Radio Frequency power amplifier [4/8],[6/7]. In lieu of reducing the peak transmitted power which affects the transmission range, few regulatory and application constraints are necessary to be implemented in the system. With the increasing peak power, the transmitter power amplifier may no longer stays in the linear region in which it is supposed to. This will
have an effect on battery’s life time. Mathematically, PAPR for any signal can be represented as,

$$\text{PAPR} = \frac{\max|x(t)|}{E[|x(t)|]}$$

(1)

where, \(x(t)\) is the signal and \(x^*(t)\) is the complex conjugate of the signal.

3. Existing scheme for PAPR reduction

![Block diagram of existing scheme for PAPR reduction](image)

3.1. Precoding technique

Let \(X\) be an input OFDM symbol with \(N\) subcarriers. One of the pre processing technique to be incorporated prior to inverse fast fourier transform and after serial to parallel conversion is precoding. The block diagram of an OFDM transmitter with precoder is seen Figure 1. The precoder is nothing but a precoding matrix \([3/4]\), which is also a unitary matrix, represented as,

\[
P = \begin{bmatrix}
P_{0,0} & P_{0,1} & \cdots & P_{0,N-1} \\
P_{1,0} & P_{1,1} & \cdots & P_{1,N-1} \\
P_{2,0} & P_{2,1} & \cdots & P_{2,N-1} \\
\vdots & \vdots & \ddots & \vdots \\
P_{L-1,0} & P_{L-1,1} & \cdots & P_{L-1,N-1}
\end{bmatrix}
\]

(3)

The output vector is obtained by multiplying the precoding matrix and input vector \(X\). The output vector \(Y\) of length \(L\) is given as

\[Y = PX = [Y_0, Y_1, Y_2, \ldots, Y_{L-1}]^T\]

(4)

The elements of the precoding matrix can be computed as follows [3/4],

\[P_{i,m} = P_{i,0} e^{-j2\pi m i / N} \text{ with}\]

\[
P_{i,0} = \begin{cases} 
\frac{(-1)^i}{\sqrt{N}}, & 0 \leq i < N_p \\
\frac{(-1)^i}{\sqrt{N}}, & N_p \leq i < N \\
\frac{(-1)^i}{\sqrt{N}} \cos \left( \frac{\pi(i-N)}{2N_p} \right), & N \leq i \leq L - 1
\end{cases}
\]

(5)

where \(L = N + N_p\) is the scrambling symbols via precoding matrix, and \(N_p\) is extra used symbols that ranges from 0 to \(N\).

3.2. Mu-Law Companding

\(\mu\)-Law is one of the simplest yet effective companding technique proposed for the peak-to-average power ratio reduction in OFDM signal. Concept of companding is in use in speech processing. The OFDM signal and speech signal are similar in some extent. Like, large signals will occur very rarely. Hence, identical companding technique has been extended to improve the transmission performance of OFDM [7/3], [8/5].

For low level inputs, the compressor characteristics are linear whereas for high level inputs, it is logarithmic. The general \(\mu\)-law compressor characteristic for different values of \(\mu\) is shown in Figure 3. From the figure, it is evident that for higher values of \(\mu\), more is the compression and for \(\mu=0\) zero compression is achieved [3/4]. A mid tread is seen at the origin and thus it holds zero value. This technique finds its application in processing of speech and music signals.
The $\mu$-law compander function can be given as
\[
H_\mu(x) = \text{sgn}(x) \left( \frac{\ln(1+\mu|x|)}{\ln(1+\mu)} \right) \text{ for } -1 \leq x \leq 1
\]  
(6)

The $\mu$-law expander function can be given as
\[
H_\mu^{-1}(y) = \text{sgn}(y) \left( \frac{1}{\mu} \right) \text{ for } -1 \leq y \leq 1
\]  
(7)

Where, $\mu$ is the companding profile. The standard value used for $\mu$ is 257.

4. The Proposed PAPR reduction technique

4.1 Combination of Pre-coding and Gamma Companding Scheme for PAPR Reduction

4.1.1. Gamma Companding

Gamma correction is used to face the entire dynamic range of amplitudes in a signal. The subcarriers with large amplitudes are left unaltered whereas subcarriers with lower amplitudes are amplified. This increases the average power of the signal thereby reducing the PAPR.

The function used in discrete-time complex OFDM symbols for gamma correction companding is given by,
\[
H_\gamma(x) = \text{A} \ \text{sgn}(x)|x|^\gamma
\]  
(8)

where $x$ represents the instantaneous value of OFDM signal and $\gamma$ represents the companding parameter. In the receiver end, in order to revert back the correction function, inverse transformation of GCC termed as GC expanding is utilized. The inverse function is given by the equation (9).
\[
H_\gamma^{-1}(y) = \text{sgn}(y) \left( \frac{1}{\gamma} \right)^{\frac{1}{\gamma}}
\]  
(9)

In gamma correction technique, fixing the appropriate value for gamma plays a significant role in PAPR reduction.

5. Results and Discussion

The specifications of OFDM system incorporated in this paper are tabulated in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>FFT size.</td>
<td>64</td>
</tr>
<tr>
<td>Number of used subcarriers.</td>
<td>52</td>
</tr>
<tr>
<td>FFT Sampling frequency</td>
<td>20MHz</td>
</tr>
<tr>
<td>Subcarrier spacing</td>
<td>312.5kHz</td>
</tr>
<tr>
<td>Used subcarrier index</td>
<td>{-26 to -1, +1 to +26}</td>
</tr>
<tr>
<td>Cyclic prefix duration</td>
<td>0.8us</td>
</tr>
<tr>
<td>Data symbol duration</td>
<td>3.2us</td>
</tr>
<tr>
<td>Total Symbol duration</td>
<td>4us</td>
</tr>
</tbody>
</table>

![Fig. 2. Compression characteristics curve](image1)

![Fig. 3. Block diagram of the combination of precoding and Gamma companding scheme in the OFDM transmitter.](image2)
The power spectrum shows the frequencies at which variations are robust and at which frequencies variations are weak. Figure 4 shows the PSD spectrum of the OFDM with precoder and Mu-Law compander. Figure 5 shows that the PSD spectrum of OFDM using precoder and gamma companding. A constant transmit spectrum within range -4 MHz to 4 MHz is seen and from -5 MHz to -4 MHz and from 4 MHz to 5 MHz, side lobes of transmit spectrum is evident. From the spectrum, we conclude that PSD level for gamma companding is better compared to Mu-Law companding. Figure 6 shows the comparative analysis of PAPR reduction performance with various companding techniques in OFDM systems.

Table 2 represents the PAPR distribution obtained for the combination of precoder and Mu-Law companding and the combination of precoder and gamma companding. We can observe that CCDF=10^0, the PAPR is reduced by approximately 3.2 dB for using gamma compander by 4.4 dB for precoder with Mu-Law companding.

6. Conclusion

High PAPR problem can be mitigated with the aid of any one of the nonlinear companding techniques owing to its simplicity and effectiveness. An approach that enjoys the advantage of precoding and gamma companding is proposed. With proper selection of precoding matrix and gamma parameter, a significant reduction in PAPR is seen. A comparative analysis on the different companding techniques in OFDM systems.
techniques is studied and the proposed technique excels other techniques in PAPR reduction and OOB emission.

References